Mark Scheme (Results)

Summer 2018

Pearson Edexcel GCE A Level Mathematics

Statistics & Mechanics (9MA0/03)

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General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the last candidate in exactly the same way as they mark the first.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification/indicative content will not be exhaustive.
- When examiners are in doubt regarding the application of the mark scheme to a candidate's response, a senior examiner must be consulted before a mark is awarded.
- Crossed out work should be marked UNLESS the candidate has replaced it with an alternative response.

PEARSON EDEXCEL GCE MATHEMATICS

General Instructions for Marking

1. The total number of marks for the paper is 100.

2. These mark schemes use the following types of marks:

- **M** marks: Method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
- **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
- **B** marks are unconditional accuracy marks (independent of M marks)
- Marks should not be subdivided.

3. Abbreviations
   These are some of the traditional marking abbreviations that will appear in the mark schemes.

   - **bod** – benefit of doubt
   - **ft** – follow through
   - the symbol $\surd$ will be used for correct ft
   - **cao** – correct answer only
   - **cso** - correct solution only. There must be no errors in this part of the question to obtain this mark
   - **isw** – ignore subsequent working
   - **awrt** – answers which round to
   - **SC**: special case
   - **o.e.** – or equivalent (and appropriate)
   - **d** or **dep** – dependent
   - **indep** – independent
   - **dp** decimal places
   - **sf** significant figures
   - ***** The answer is printed on the paper or ag- answer given

4. All M marks are follow through.

   A marks are 'correct answer only' (cao.), unless shown, for example, as A1 ft to indicate that previous wrong working is to be followed through. After a misread however, the subsequent A marks affected are treated as A ft, but answers that don't logically make sense e.g. if an answer given for a probability is >1 or <0, should never be awarded A marks.

5. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

6. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response.
   If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.

7. Ignore wrong working or incorrect statements following a correct answer.

8. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used. If no such alternative answer is provided but the response is deemed to be valid, examiners must escalate the response for a senior examiner to review.

Section B: MECHANICS

Question	Scheme	Marks	AOs
6.	Integrate **v** w.r.t. time	M1	1.1a
	$\mathbf{r} = 2t^{\frac{1}{2}}\mathbf{i} - 2t^2\mathbf{j}$ $(+\ \mathbf{C})$	A1	1.1b
	Substitute $t = 4$ and $t = 1$ into their **r**	M1	1.1b
	$t = 4, \mathbf{r} = 4\mathbf{i} - 32\mathbf{j}(+\ \mathbf{C})$; $t = 1, \mathbf{r} = 2\mathbf{i} - 2\mathbf{j}(+\ \mathbf{C})$ or $(4, -32)$; $(2, -2)$	A1	1.1b
	$\sqrt{2^2 + (-30)^2}$	M1	1.1b
	$\sqrt{904} = 2\sqrt{226}$	A1	1.1b
		(6)	
			(6 marks)

Notes: Allow column vectors throughout

M1: At least one power increasing by 1.

A1: Any correct (unsimplified) expression

M1: Must have attempted to integrate **v**. Substitute $t = 4$ and $t = 1$ into their **r** to produce 2 vectors (or 2 points if just working with coordinates).

A1: $4\mathbf{i} - 32\mathbf{j}(+\ \mathbf{C})$ and $2\mathbf{i} - 2\mathbf{j}(+\ \mathbf{C})$ or $(4, -32)$ and $(2, -2)$. These can be seen or implied.

M1: Attempt at distance of form $\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$ for their points. Must have 2 non zero terms.

A1: $\sqrt{904} = 2\sqrt{226}$ or any equivalent surd (exact answer needed)

Question	Scheme	Marks	AOs
7(a)	Resolve vertically	M1	3.1b
	$R + 40\sin\alpha = 20g$	A1	1.1b
	Resolve horizontally	M1	3.1b
	$40\cos\alpha - F = 20a$	A1	1.1b
	$F = 0.14R$	B1	1.2
	$a = 0.396$ or 0.40 (m s^{-2})	A1	2.2a
		(6)	
(b)	Pushing will increase R which will increase available F	B1	2.4
	Increasing F will <u>decrease a</u> * GIVEN ANSWER	B1*	2.4
		(2)	
			(8 marks)

Notes:

(a)

M1: Resolve vertically with usual rules applying

A1: Correct equation**.** Neither g nor $\sin a$ need to be substituted

M1: Apply $F = ma$ horizontally, with usual rules

A1: Neither F nor $\cos a$ need to be substituted

B1: $F = 0.14R$ seen (e.g. on a diagram)

A1: Either answer

(b)

B1: Pushing increases R which produces an increase in available (limiting) friction

B1: F increase produces an <u>a decrease (need to see this)</u>

N.B. It is possible to score B0 B1 but for the B1, some "explanation" is needed to say why friction is increased e.g. by pushing into the ground.

Question	Scheme	Marks	AOs
8(a)	Use of $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$: $(7\mathbf{i} - 10\mathbf{j}) = 2(2\mathbf{i} - 3\mathbf{j}) + \frac{1}{2}\mathbf{a}2^2$	M1	3.1b
	$\mathbf{a} = (1.5\mathbf{i} - 2\mathbf{j})$	A1	1.1b
	$\lvert\mathbf{a}\rvert = \sqrt{1.5^2 + (-2)^2}$	M1	1.1b
	= 2.5 m s^{-2} * GIVEN ANSWER	A1*	2.1
		(4)	
(b)	Use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t = (2\mathbf{i} - 3\mathbf{j}) + 2(1.5\mathbf{i} - 2\mathbf{j})$	M1	3.1b
	$= (5\mathbf{i} - 7\mathbf{j})$	A1	1.1b
	$\mathbf{v} = (5\mathbf{i} - 7\mathbf{j}) + t(4\mathbf{i} + 8.8\mathbf{j}) = (5 + 4t)\mathbf{i} + (8.8t - 7)\mathbf{j}$ and $(5 + 4t) = (8.8t - 7)$	M1	3.1b
	t = 2.5 (s)	A1	1.1b
		(4)	
			(8 marks)

Notes: Allow column vectors throughout

(a)

No credit for individual component calculations

M1: Using a complete method to obtain the acceleration. **N.B.** Equation, in **a** only, could be obtained by two integrations

ALTERNATIVE

M1: Use velocity at half-time (t = 1) = Average velocity over time period

So at t = 1, $\mathbf{v} = \frac{1}{2}(7\mathbf{i} - 10\mathbf{j})$ so $\mathbf{a} = \frac{1}{2}(7\mathbf{i} - 10\mathbf{j}) - (2\mathbf{i} - 3\mathbf{j})$

N.B. could see $(7\mathbf{i} - 10\mathbf{j}) = (4\mathbf{i} - 6\mathbf{j}) + 2\mathbf{a}$ as first line of working

A1: Correct **a** vector

M1: Attempt to find magnitude of their **a** using form $\sqrt{a^2 + b^2}$

A1*: Correct GIVEN ANSWER obtained correctly

(b)

M1: Using a complete method to obtain the velocity at *A* e.g.by use of $\mathbf{v} = \mathbf{u} + \mathbf{a}t$ with t = 2 and **u** = 2**i** – 3**j** and their **a**

OR: by use of $\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

OR: by integrating their **a,** with addition of **C** = 2**i** – 3**j,** and putting t = 2

A1: correct vector

M1: Complete method to find equation in t only

e.g. by using $\mathbf{v} = \mathbf{u} + \mathbf{a}t$, with their **u** and equating **i** and **j** components

OR: by integrating $(4\mathbf{i} + 8.8\mathbf{j})$, with addition of a constant, and equating **i** and **j** components.

N.B. Must be equating **i** and **j** components of a velocity vector and must be their velocity at *A*, to give an equation in *t* only for this M mark

A1: 2.5 (s)

Question	Scheme	Marks	AOs
9(a)	Moments about A (or any other complete method)	M1	3.3
	$T2a\sin a = Mga + 3Mgx$	A1	1.1b
	$T = \frac{Mg(a+3x)}{2a\times\frac{3}{5}} = \frac{5Mg(3x+a)}{6a}$ * GIVEN ANSWER	A1*	2.1
		(3)	
(b)	$\frac{5Mg(3x+a)}{6a}\cos a = 2Mg$ **OR** $2Mg.2a\tan\alpha = Mga + 3Mgx$	M1	3.1b
	$x = \frac{2a}{3}$	A1	2.2a
		(2)	
(c)	Resolve vertically **OR** Moments about B	M1	3.1b
	$Y = 3Mg + Mg - \frac{5Mg(3.\frac{2a}{3}+a)}{6a}\sin a$ $2aY = Mga + 3Mg(2a - \frac{2a}{3})$ **Or**: $Y = 3Mg + Mg - \left(\frac{2Mg}{\cos\alpha}\right)\sin\alpha$	A1**ft**	1.1b
	$Y = \frac{5Mg}{2}$ **N.B.** May use $R\sin\beta$ for Y and/or $R\cos\beta$ for X throughout	A1	1.1b
	$\tan\beta = \frac{Y}{X}$ or $\frac{R\sin\beta}{R\cos\beta} = \frac{\frac{5Mg}{2}}{2Mg}$	M1	3.4
	$= \frac{5}{4}$	A1	2.2a
		(5)	
(d)	$\frac{5Mg(3x+a)}{6a} \le 5Mg$ and solve for x	M1	2.4
	$x \le \frac{5a}{3}$	A1	2.4
	For rope not to break, block can't be more than $\frac{5a}{3}$ from A oe Or just: $x \le \frac{5a}{3}$, if no incorrect statement seen. **N.B. If the correct inequality is not found,** their comment must mention 'distance from A'.	B1 **A1**	2.4
		(3)	
			(13 marks)

Notes:

(a)

M1: Using M(*A*), with usual rules, or any other complete method to obtain an equation in *a*, *M*, *x* and *T* only.

A1: Correct equation

A1*: Correct PRINTED ANSWER, correctly obtained, need to see $\sin\alpha = \frac{3}{5}$ used.

(b)

M1: Using an appropriate strategy to find *x*. e.g. Resolve horizontally with usual rules applying OR Moments about *C*. Must use the given expression for *T*.

A1: Accept 0.67*a* or better

(c)

M1: Using a complete method to find *Y* (or *R*sin β) e.g. resolve vertically or Moments about *B*, with usual rules

A1 ft: Correct equation with their *x* substituted in *T* expression or using $T = \frac{2Mg}{\cos\alpha}$

A1: *Y* (or *R*sin β) = $\frac{5Mg}{2}$ or 2.5*Mg* or 2.50*Mg*

M1: For finding an equation **in tan** β **only** using $\tan\beta = \frac{Y}{X}$ or $\tan\beta = \frac{X}{Y}$

This is independent but must have found a *Y*.

A1: Accept $\frac{-5}{4}$ if it follows from their working.

(d)

M1: Allow *T* = 5*Mg* or *T* < 5*Mg* and solves for *x*, showing all necessary steps (M0 for *T* > 5*Mg*)

A1: Allow $x = \frac{5a}{3}$ or $x < \frac{5a}{3}$. Accept 1.7*a* or better.

B1: Treat as A1. For any appropriate equivalent fully correct comment or statement. E.g. maximum value of *x* is $\frac{5a}{3}$

Question	Scheme	Marks	AOs
10(a)	Using the model and vertical motion: $0^2 = (U\sin\alpha)^2 - 2g\times(3-2)$	M1	3.3
	$U^2 = \dfrac{2g}{\sin^2\alpha}$ * GIVEN ANSWER	A1*	2.2a
		(2)	
(b)	Using the model and horizontal motion: $s = ut$	M1	3.4
	$20 = Ut\cos\alpha$	A1	1.1b
	Using the model and vertical motion: $s = ut + \dfrac{1}{2}at^2$	M1	3.4
	$-\dfrac{5}{4} = Ut\sin\alpha - \dfrac{1}{2}gt^2$	A1	1.1b
	sub for t: $-\dfrac{5}{4} = U\sin\alpha\left(\dfrac{20}{U\cos\alpha}\right) - \dfrac{1}{2}g\left(\dfrac{20}{U\cos\alpha}\right)^2$	M1 (I)	3.1b
	sub for U^2	M1(II)	3.1b
	$-\dfrac{5}{4} = 20\tan\alpha - 100\tan^2\alpha$	A1(I)	1.1b
	$(4\tan\alpha - 1)(100\tan\alpha + 5) = 0$	M1(III)	1.1b
	$\tan\alpha = \dfrac{1}{4} \Rightarrow \alpha = 14^0$ or better	A1(II)	2.2a
		(9)	
	N.B. For the last 5 marks, they may set up a quadratic in t, by substituting for $U\sin\alpha$ first, then solve the quadratic to find the value of t, then use $20 = Ut\cos\alpha$ to find α. The marks are the same but earned in a different order. Enter on ePen in the corresponding M and A boxes above, as indicated below.		
	Sub for $U\sin\alpha$ to give equation in t only	M1(II)	
	$-\dfrac{5}{4} = \sqrt{2g}t - \dfrac{1}{2}gt^2$	A1(I)	
	Solve for t	M1(III)	
	$t = \dfrac{5}{\sqrt{2g}}$ or 1.1 or 1.13 and use $20 = Ut\cos\alpha$	M1(I)	
	$\alpha = 14^0$ or better	A1(II)	
(b)	**ALTERNATIVE**		

	Using the model and horizontal motion: $s = ut$	M1	3.4
	$20 = Ut\cos\alpha$	A1	1.1b
	A to top: $s = vt - \frac{1}{2}at^2$ and top to T: $s = ut + \frac{1}{2}at^2$		
	$1 = \frac{1}{2}gt_1^2 \Rightarrow t_1 = \sqrt{\frac{2}{g}}$ and $\frac{9}{4} = \frac{1}{2}gt_2^2 \Rightarrow t_2 = \frac{3}{\sqrt{2g}}$ Total time $t = t_1 + t_2$	M1	3.4
	$= \sqrt{\frac{2}{g}} + \frac{3}{\sqrt{2g}} \quad (= \frac{5}{\sqrt{2g}})$	A1	1.1b
	$20 = U\frac{5}{\sqrt{2g}}\cos\alpha$ (sub. for t)	M1	3.1b
	$20 = \sqrt{\frac{2g}{\sin^2\alpha}}\frac{5}{\sqrt{2g}}\cos\alpha$ (sub. for U)	M1	3.1b
	$\tan\alpha = \frac{1}{4}$	A1	1.1b
	Solve for α	M1	1.1b
	$\Rightarrow \alpha = 14^0$ or better	A1	2.2a
		(9)	
(c)	The target will have dimensions so in practice there would be a range of possible values of α **Or** There will be air resistance **Or** The ball will have dimensions **Or** Wind effects **Or** Spin of the ball	B1	3.5b
		(1)	
(d)	Find U using their α e.g. $U = \sqrt{\frac{2g}{\sin^2\alpha}}$	M1	3.1b
	Use $20 = Ut\cos\alpha$ (or use vertical motion equation)	A1 **M1**	1.1b
	$t = \frac{5}{\sqrt{2g}}$ or 1.1 or 1.13	B1 **A1**	1.1b
		(3)	
(d)	**ALTERNATIVE**		

	A to top: $s = vt - \frac{1}{2}at^2$ and top to T: $s = ut + \frac{1}{2}at^2$	M1	3.1b
	$1 = \frac{1}{2}gt_1^2 \Rightarrow t_1 = \sqrt{\frac{2}{g}}$ and $\frac{9}{4} = \frac{1}{2}gt_2^2 \Rightarrow t_2 = \frac{3}{\sqrt{2g}}$ Total time $t = t_1 + t_2$	A1 **M1**	1.1b
	$= = \sqrt{\frac{2}{g}} + \frac{3}{\sqrt{2g}}$ $(= \frac{5}{\sqrt{2g}})$ = 1.1 or 1.13 (s)	B1 **A1**	1.1b
		(3)	
			(15 marks)

Notes:

(a)

M1: Or any other complete method to obtain an equation in U, g and α **only**

A1*: Correct GIVEN ANSWER

(b)

M1: Using horizontal motion

A1: Correct equation

M1: Using vertical motion . N.B. M0 if they use $s = \pm 2$ or ± 3, but allow $s = \pm 1.25$ or ± 0.75 or ± 2.25 or ± 2.75

A1: Correct equation

M1: Using $20 = Ut\cos\alpha$ to sub. for t

M1: Substituting for U^2 using (a)

A1: Correct quadratic equation (in $\tan\alpha$ **or** $\cot\alpha$ **)**

M1: Solve a 3 term quadratic, either by factorisation or formula (or by calculator (implied) if answer is correct) **and find** α

A1: $\alpha = 14^o$ or better (No restriction on accuracy since g's cancel)

N.B. If answer is correct, previous M mark can be implied, but if answer is incorrect, an explicit attempt to solve must be seen to earn the previous M mark.

(b) ALTERNATIVE

M1: Using the model with the usual rules applying to the equation

A1: Correct equation

M1: Using the model to obtain the **total** time from A to T

A1: Correct **total** time t

M1: Substitute for t in $20 = Ut\cos\alpha$

M1: Substitute for U in $20 = Ut\cos\alpha$, using part (a)

A1: Correct equation in $\tan\alpha$ **only**

M1: Solve equation for α

A1: $\alpha = 14^o$ or better (No restriction on accuracy since g's cancel)

N.B. If they quote the equation of the trajectory $y = x\tan\alpha - \frac{gx^2}{2U^2\cos^2\alpha}$ oe **AND** put in values for *x* and *y*, could score first 5 marks, M1A1M1A1M1 (nothing for the equation only); wrong *x* value loses first A mark and wrong *y* value loses second A mark

(c)

B1: Give one limitation of the model e.g. the ball will have dimensions, or there will be air resistance or wind effects or spin

N.B. B0 if any incorrect extra(s) but ignore extra consequences.

(d)

M1: Using their a to find a value for *U*

A1: Treat as M1: Using their *U* to find a value for *t*

B1: Treat as A1 : *t* = 1.1 or 1.10 (since depends on *g* = 9.8)

(d) ALTERNATIVE

M1: Using their a to find a value for *U*

A1: Treat as M1: Using their *U* to find a value for *t*

B1: Treat as A1 : *t* = 1.1 or 1.10 (since depends on *g* = 9.8)

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